Section 1.5: Sets

Throughout, assume that $A, B, C \subset X$.

1. (a) Prove DeMorgan’s Laws:
   
   $$(A \cup B)^c = A^c \cap B^c, \text{ and } (A \cap B)^c = A^c \cup B^c.$$ 

   (b) How can we generalize this result to an arbitrary collection of sets? State and prove the theorem.

2. Prove the distributivity properties:
   
   $$A \cap (B \cup C) = (A \cap B) \cup (A \cap C) \text{ and } A \cup (B \cap C) = (A \cup B) \cap (A \cup C).$$

3. The symmetric difference of two sets is defined as
   
   $$A \Delta B = (A \setminus B) \cup (B \setminus A).$$

   Prove that the symmetric difference is associative:
   
   $$(A \Delta B) \Delta C = A \Delta (B \Delta C).$$