COUNTEREXAMPLES AND INVALIDITY

I. The Famous Forms Method

A. Review

B. Exercises

C. Limitations

1. There are numerous valid argument forms. Our method recognizes only five. So our method does not show them valid.

2. Our method does not show invalidity.

II. Counterexamples and Argument Forms

Consider:

*Argument 1*

1. If there is a God, then life has meaning.
2. Life has meaning.
3. So, there is a God.

Its form is popular enough to have a name:

*The Fallacy of Affirming the Consequent*

1. If A, then B.
2. B.
3. So, A

This is an *invalid argument form*, i.e. it has some substitution instances that are invalid. You can see this by way of a *good counterexample*.

- **Def.** A *good counterexample* to an argument form is a substitution instance in which the premises are well-known truths and the conclusion is a well-known falsehood.

For example:
Argument 2
1. If [A] there is fire in our room, then [B] there is air in our room. (well-known truth)
2. [B] There is air in our room. (well-known truth)
3. So, [A] there is fire in our room. (well-known falsehood)

Argument 2 is a substitution instance of the Fallacy of Affirming the Consequent. Moreover, Argument 2 is clearly invalid. So, the Fallacy of Affirming the Consequent is an invalid form.

Consider

Argument 3
1. If there is an omnipotent, omniscient, and morally perfect creator, then there is a God.
2. There is no omnipotent, omniscient, and morally perfect creator.
3. So, there is no God.

Its form is popular enough to have a name:

The Fallacy of Denying the Antecedent
1. If A, then B.
2. not-A.
3. So, not-B.

This too is an invalid argument form. Here’s a good counterexample.

Argument 4
1. If [A] there is fire in our room, then [B] there is air in our room. (well-known truth)
2. [not-A] There is no fire in our room. (well-known truth)
3. So, [not-B] there is no air in our room. (well-known falsehood)

Argument 4 is a substitution instance of the Fallacy of Denying the Antecedent. Moreover, Argument 4 is clearly invalid. So, the Fallacy of Denying the Antecedent is an invalid form.

Upshot: Counterexamples can be used to show the invalidity of argument forms.
III. Counterexamples and Arguments

A. Question

How is showing a counterexample to an invalid form relevant to showing that an argument is invalid?

B. Answer

If you can identify the most logically sensitive form of an argument—by paying due attention to its key logical terms, e.g. “if-then,” “either-or,” “and,” “not,” and more besides—and if you can give a counterexample to that form, then you will have shown that the argument is invalid. For you will have shown that it is possible for the premises to be true while the conclusion is false.

C. Explanation

Every argument has a form that is invalid. Illustration:

<table>
<thead>
<tr>
<th>Argument 5</th>
<th>Form</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. If Fritz is a cat, then Fritz is a mammal.</td>
<td>1. A.</td>
</tr>
<tr>
<td>2. Fritz is a cat.</td>
<td>2. B.</td>
</tr>
<tr>
<td>3. So, Fritz is a mammal.</td>
<td>3. So, C.</td>
</tr>
</tbody>
</table>

But that does not mean that Argument 5 is invalid. For it has a more logically sensitive form which is valid, one that is sensitive to the key logical terms, particularly “if-then”:

<table>
<thead>
<tr>
<th>Modus Ponens</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. If A, then B.</td>
</tr>
<tr>
<td>2. A.</td>
</tr>
<tr>
<td>3. So, B.</td>
</tr>
</tbody>
</table>

D. Moral

All it takes for an argument to be valid is for it to be a substitution instance of one valid argument form. That’s enough. It might well be a substitution instance of some invalid argument form. But that does not mean that the argument is invalid. Only when the most logically sensitive form of an argument is invalid does it mean that the argument is invalid.
IV. The Counterexample Method

1. Identify the most logically sensitive form of the argument. Use capital letters to stand for statements.

2. Find English statements that, if substituted for the capital letters in the conclusion of the argument form, produce a well-known falsehood.

3. Substitute these English statements for the relevant capital letters uniformly throughout the rest of the argument.

4. Find English statements that, if substituted uniformly for the remaining capital letters in the argument form, produce premises that are well-known truths.

5. Check your work. If you have succeeded, you have shown the argument to be invalid.

Note: if you combine the Famous Form Method and the Counterexample Method, you will have a combined method that is more useful than either alone.
V. Exercises

1.3 Part A

*FORM*       *GOOD COUNTEREXAMPLE*

(1)
1. If A, then B.  1. If Barack Obama is a whale, then he is an animal. (WKT)
2. Not-A.        2. Barack Obama is not a whale. (WKT)
3. So, not-B.     3. So, Barack Obama is not an animal. (WKF)

(2)
1. If A, then B.  1. If Barack Obama is a whale, then he is an animal. (WKT)
2. B.            2. Barack Obama is an animal. (WKT)
3. So, A.        3. So, Barack Obama is a whale. (WKF)

(3)
1. If A, then B.  1. If Barack Obama is a whale, then he is animal. (WKT)
2. So, A.        2. So, Barack Obama is a whale. (WKF)

(4)
1. If A, then B.  1. If Obama is a whale, then he is an animal. (WKT)
2. So, if not-A, then not-B.  2. So, if Obama is not a whale, then he is not an animal. (WKF)

(10)
1. If A, then B.  1. If Obama is a whale, then he has fins. (WKT)
2. Not-B.        2. Obama does not have fins. (WKT)
3. So, A.        3. So, Obama is a whale. (WKF)

(15)
1. Either A or B.  1. Either Obama is a human or he is a male. (WKT—remember: inclusive disjunction!)
2. A.            2. Obama is a human. (WKT)
3. So, not-B.     3. So, Obama is not a male. (WKF)