I. Nonparametric ANOVA — Kruskal-Wallis test (\(\cong\) ANOVA by ranks)

A. Application:
- whenever parametric single-factor ANOVA appropriate,
- when parametric parametric assumptions violated: non-normal populations; \(\neq\) variances

B. Assumption: sampled populations have same dispersions & shapes

C. Hypotheses:
- \(H_0:\) measurements the same in all \(k\) populations
- \(H_A:\) measurements not the same in all \(k\) populations

D. Procedure: Rank data — same as with Mann-Whitney two-sample test

E. Test statistic:
\[
H = \frac{12}{N(N+1)} \sum_{i=1}^{k} \frac{R_i^2}{n_i} - 3(N+1)
\]
where \(n_i = \#\) in group \(i\); \(R_i = \) sum ranks in \(i\)

F. Compare w/ critical values, Table B.13

G. If \(k > 5\) or \(n_i\) large (> 6 or 7), approximate critical values of \(H:\) \(\chi^2\) w/ \(k-1\) DF

H. If tied ranks, \(H\) an underestimate. Correction factor:
\[
C = 1 - \frac{\sum t}{N^2 - N}
\]
\[
H_c = \frac{H}{C}
\]
\[
\sum t = \sum_{i=1}^{m} (t_i^3 - t_i)
\]
\(t_i = \#\) ties in \(i^{th}\) group, \(m = \#\) groups of tied ranks

\(H_c \approx H\) when \(t_i's << N\)

II. Test for Homogeneity of Variances — Bartlett’s test

A. Hypotheses:
- \(H_0:\) \(\sigma_1^2 = \sigma_2^2 = \ldots = \sigma_k^2\)
- \(H_A:\) variances are not all equal

B. Test Statistic:
\[
B = (\ln s_p^2) \left( \sum_{i=1}^{k} v_i - \sum_{i=1}^{k} v_i \ln s_i^2 \right)
\]
\(v_i = n_i - 1,\) \(s_p^2 =\) pooled variance = \(\sum SS_i / \sum v_i\)

C. \(B \approx \chi^2\) critical value: \(\chi^2_{\alpha, k-1}\), where \(k = \) number of samples

E. More accurate w/ correction factor:
\[
C = 1 + \frac{1}{3(k-1)} \left( \sum \frac{1}{v_i} - \frac{1}{\sum v_i} \right)
\]
Corrected statistic: \(B_c = B/C\) \(\) Compare \(B_c\) with \(\chi^2_{\alpha, k-1}\)

F. Test badly affected by deviations from normality
   Not recommended to test assumptions prior to doing single-factor ANOVA,
   – esp. since ANOVA relatively robust to unequal variances.