1 Chi-Square Test for Goodness of Fit

1.1 Hypotheses:
- \( H_0 \): sample was taken from a population with proportions \( p_1, p_2, \ldots \)
- \( H_A \): sample was taken from a population with proportions different from \( p_1, p_2, \ldots \)

1.2 Test Procedure:
1.2.1 Arrange data into frequencies observed for each category.
1.2.2 Calculate frequencies expected for sample size \( n \) if \( H_0 \) true.
1.2.3 Calculate chi-square statistic:
\[
\chi^2 = \sum_{i=1}^{k} \frac{(f_i - \hat{f}_i)^2}{\hat{f}_i}
\]
- \( f_i \) = frequency observed in category \( i \)
- \( \hat{f}_i \) = frequency expected in category \( i \) if \( H_0 \) true

1.2.4 Compare with critical value, \( \chi^2_{a,\nu} \), degrees of freedom: \( \nu = k - 1 \)

1.2.5 Reject \( H_0 \) if \( \chi^2 \geq \chi^2_{a,\nu} \)

1.3 Example: gender composition of ESCI 340: 26 students; 11 women, 15 men.
- gender composition of WWU undergraduates: 54.5% women; 45.5% men
- "expected" gender composition of ESCI 340: 14.2 women, 11.8 men.
\[
\chi^2 = \frac{(11 - 14.2)^2}{14.2} + \frac{(15 - 11.8)^2}{11.8} = 0.709 + 0.849 = 1.559
\]
\( \nu = k - 1 = 2 - 1 = 1 \)
\( \chi^2_{0.05,1} = 3.841 \)

Do not reject \( H_0 \) (0.25 > \( P > 0.10 \))

1.4 Chi-Square Correction for Continuity
\[
\chi^2_c = \sum_{i=1}^{k} \left( \frac{|f_i - \hat{f}_i| - 0.5}{\hat{f}_i} \right)^2
\]
do not use when \( k > 2 \)

Application to ESCI 340 gender composition:
\[
\chi^2_c = \left( \frac{|11 - 14.2| - 0.5}{14.2} \right)^2 + \left( \frac{|15 - 11.8| - 0.5}{11.8} \right)^2
\]
\( = 0.503 + 0.603 = 1.106 \)

Do not reject \( H_0 \) (0.50 > \( P > 0.25 \))

2 Komogorov-Smirnov Goodness of Fit Test for Discrete Data

1.1 Data sorted into categories.

1.2 Same hypotheses as above.

1.3 Expected frequencies \( \hat{f}_i \) calculated as in Chi-squared goodness of fit test.

1.4 Calculate cumulative observed frequencies \( F_i \) and cumulative expected frequencies \( \hat{F}_i \):
- cumulative frequency for \( i \) is sum of frequencies 1 through \( i \).
2 Komogorov-Smirnov Goodness of Fit Test (continued)

1.5 For each category, \( i \), determine absolute difference:

\[ |d_i| = |F_i - \hat{F}_i| \]

1.6 Test statistic, \( d_{\text{max}} \), is largest \( |d_i| \).

Note: critical value depends on sample size \( (n) \) and number of categories \( (k) \).

1.7 Kolmorgorov-Smirnov test more powerful than chi-square test when \( n \) small or \( \hat{F}_i \) values small.

1.8 Example: Grizzly bear age distribution in Greater Yellowstone Ecosystem.

\( H_0 \): The age distribution of female Yellowstone grizzly bears prior to ESA listing was equal to the proportions of the stable age distribution: cubs of the year (0.198), yearlings (0.152), two year-olds (0.123), three year-olds (0.099), and adults (0.428).*

The Craighead brothers† reported age structure data on the 83 GYE female grizzly bears prior to ESA listing ("observed frequency," below).

<table>
<thead>
<tr>
<th>Age</th>
<th>Cubs</th>
<th>1yr</th>
<th>1yr</th>
<th>3yr</th>
<th>Adults</th>
</tr>
</thead>
<tbody>
<tr>
<td>Observed frequency (( f_i ))</td>
<td>14</td>
<td>8</td>
<td>8</td>
<td>6</td>
<td>47</td>
</tr>
<tr>
<td>Expected frequency (( \hat{f}_i ))</td>
<td>16.4</td>
<td>12.6</td>
<td>10.2</td>
<td>8.2</td>
<td>35.5</td>
</tr>
<tr>
<td>Cumulative obs.freq.(( F_i ))</td>
<td>14</td>
<td>22</td>
<td>30.</td>
<td>36</td>
<td>83</td>
</tr>
<tr>
<td>Cumulative exp.freq. (( \hat{F}_i ))</td>
<td>16.4</td>
<td>29.1</td>
<td>39.3</td>
<td>47.5</td>
<td>83</td>
</tr>
<tr>
<td>(</td>
<td>d_i</td>
<td>=</td>
<td>F_i - \hat{F}_i</td>
<td>)</td>
<td>2.4</td>
</tr>
</tbody>
</table>

\[ d_{\text{max}} = 11.5 \]

From Zar Table B.8, \( (d_{\text{max}})_{0.05,5,83} = 11 \) and \( (d_{\text{max}})_{0.02,5,83} = 12 \).

Conclude that age proportions of GYE grizzly bears differed from the stable age distribution prior to delisting \((0.02 < P < 0.05)\). Younger bears were disproportionately less abundant and adult bears were disproportionately more abundant than the stable age distribution.


3 Chi-Square Analysis of Independence (Contingency Tables)

3.1 Hypotheses:

\( H_0 \): In sampled population, factors are independent. \( H_A \): Factors are not independent.

3.2 Test Procedure:

\[ \chi^2 = \sum \sum \frac{(f_{ij} - \hat{f}_{ij})^2}{\hat{f}_{ij}} \quad f_{ij} = \text{frequency observed in row } i \& \text{ column } j \]

\[ \hat{f}_{ij} = \# \text{ expected in row } i, \text{ column } j \text{ if } H_0 \text{ true} \]

\[ \hat{f}_{ij} = \frac{(R_i)(C_j)}{n} \quad \text{DF} = (r - 1)(c - 1) \]