1 Intro to Statistical Hypothesis Testing

1.1 Inference about population mean(s)

1.2 Null Hypothesis ($H_0$): No real difference (association, effect, etc.);
   $\rightarrow$ observed difference in samples is due to chance alone

1.3 Alternative hypothesis ($H_A$): $H_0$ & $H_A$ must account for all possible outcomes
e.g., $H_0$: $\mu = \mu_0$, $H_A$: $\mu \neq \mu_0$

1.4 State hypotheses before collecting data!

1.5 Typical procedure:
   1.5.1 State/clarify the research question
   1.5.2 Translate the question into statistical hypotheses
   1.5.3 Select a significance level ($\alpha$)
   1.5.4 Collect data (e.g., random sample)
   1.5.5 Look at, plot data; check for errors, evaluate distributions, etc.
   1.5.6 Select appropriate test
   1.5.7 Calculate sample(s) mean(s), standard deviation(s), standard error(s)
   1.5.8 Calculate the test statistic, e.g., $t_{\text{calc}}$
   1.5.9 Determine probability (P-value):
      If $H_0$ true, probability of sample mean at least as far from $\mu$ as $X$
   1.5.10 If $P < \alpha$, reject $H_0$ and accept $H_A$.
      Otherwise indeterminate result (neither accept nor reject $H_0$).
   1.5.11 Answer the research question

2 The Distribution of Means

2.1 Central Limit Theorem: random samples (size $n$) drawn from population
   $\rightarrow$ sample means will become normal as $n$ gets large (in practice, $n \geq 20$)

2.2 Variance of sample means $\downarrow$ as $\uparrow n$:
   $\sigma^2_{\bar{x}} = \frac{\sigma^2}{n}$  $s^2_{\bar{x}} = \frac{s^2}{n}$  $s_{\bar{x}} = \frac{s}{\sqrt{n}}$
   $\sigma_{\bar{x}}^2$ = variance of the pop. mean
   $\sigma_{\bar{x}}$ = standard error
   $s_{\bar{x}}$ = sample standard error

3 Types of Errors

3.1 Type I error: incorrect rejection of true null hypothesis (Probability = $\alpha$)

3.2 Type II error: failure to reject false null hypothesis (Probability = $\beta$)

3.3 Two other possibilities: (1) do not reject true null hypothesis; (2) reject false null hypothesis

3.4 Significance level = probability of type I error (= $\alpha$)
   Must state significance level before collecting data!

3.5 In scientific communication, restrict “significant” to statistical context;
   never use “significant” as synonym for “important” or “substantial”

3.6 Industrial statistics: $\alpha$ called “producer’s risk” = $P${reject good ones}$
   $\beta$ called “consumer’s risk”, $P${accept bad ones}
4 Hypothesis Tests Concerning the Mean – Two-Tailed

4.1 Unknown \( \sigma^2 \): use t-distribution (t) instead of Normal (z):  
\[
t = \frac{\bar{X} - \mu_0}{s_X}
\]

4.2 Performing the \( t \)-test

4.2.1 State null (\( H_0 \)) & alternative (\( H_A \)) hypotheses: e.g., \( H_0: \mu = 0 \), \( H_A: \mu \neq 0 \)

4.2.2 State significance level (\( \alpha \)); e.g., \( \alpha = 0.05 \)

4.2.3 Define critical region; e.g., 2-tailed test: if \( P(|t_{\text{calc}}|) \leq 0.05 \), then reject \( H_0 \)
  i.e., if \( |t_{\text{calc}}| \geq t_{\alpha(2),\nu} \), then reject \( H_0 \)
  e.g.: one sample, 2-tailed test, w/ \( \alpha = 0.05 \), and \( n=25 \) (\( \nu = 24 \)): \( t_{\alpha(2),\nu} = t_{0.05(2),24} = 2.064 \)

4.2.4 Determine \( \bar{X}, s_X \); e.g., \( \bar{X} = 5.0, s_X = 2.0 \)

4.2.5 Calculate \( t_{\text{calc}} \):  
\[
t = \frac{\bar{X} - \mu_0}{s_X}
\]
  e.g., \( t = \frac{5.0 - 0}{2.0} = 2.5 \)

4.2.6 Find \( t_{\text{critical}} \) (= \( t_{\alpha(2),\nu} \)) in \( t \)-table (Zar table B.3)

4.2.7 If \( t_{\text{calc}} \geq t_{\text{critical}} \), then reject \( H_0 \); otherwise do not reject \( H_0 \)
  e.g., for \( \alpha = 0.05 \), and \( n=25 \) (\( \nu = 24 \)) \( t = 2.5 > 2.064 \) → reject \( H_0 \), conclude that \( \mu \neq 0 \)

4.3 Cannot test hypotheses about single observation (\( \nu = n-1 = 1-1 = 0 \))

4.4 Assumptions of one sample \( t \)-test:
  1. data are a random sample
  2. sample from pop. with normal distribution

4.5 Replication:
  measurements must be truly replicated; avoid pseudoreplication

5 One-Tailed Tests

5.1 Two-tailed hypotheses: \( H_0: \mu = \mu_0 \), \( H_A: \mu \neq \mu_0 \)
  Difference could be positive or negative

5.2 One-tailed hypotheses: \( H_0: \mu \leq \mu_0 \), \( H_A: \mu > \mu_0 \)

5.3 Critical value for one-tailed test always smaller than for two-tailed (easier to get significance)
  e.g., for \( \alpha = 0.05 \), \( Z_{\alpha(1)} = 1.645 \) and \( Z_{\alpha(2)} = 1.960 \)
  must declare hypotheses before examining data

5.4 If \( t \geq t_{\alpha(1),\nu} \) then reject \( H_0 \)
6 Confidence Limits of the Mean

6.1 t-distribution: indicates fraction of all possible sample means greater (or less than) t
where \( t = \frac{\bar{X} - \mu}{s_{\bar{X}}} \)

6.2 95% of all t-values occur between \( t_{\alpha(2)\nu} \) and \( t_{\alpha(2)\nu} \)
\[
P \left[ -t_{0.05(2),\nu} \leq \frac{\bar{X} - \mu}{s_{\bar{X}}} \leq t_{0.05(2),\nu} \right] = 0.95
\]

6.3 Solve for \( \mu \):
\[
P \left[ \bar{X} - t_{0.05(2),\nu}s_{\bar{X}} \leq \mu \leq \bar{X} + t_{0.05(2),\nu}s_{\bar{X}} \right] = 0.95
\]

6.4 95% Confidence limits of the mean:
- Lower limit: \( \bar{X} - t_{0.05(2),\nu}s_{\bar{X}} \)
- Upper limit: \( \bar{X} + t_{0.05(2),\nu}s_{\bar{X}} \)
- Concise statement: \( \bar{X} \pm t_{0.05(2),\nu}s_{\bar{X}} \)

6.5 General notation: 2-tailed, with sample size \( n-1 \), @ significance level \( \alpha \):
\[
\bar{X} \pm t_{\alpha(2),\nu}s_{\bar{X}} \quad \rightarrow \quad \text{e.g., 99% confidence interval}
\]

6.6. Reporting variability about the mean
In table, figure, text, must show/state (& look for) 4 things:
(1) value of mean
(2) units of measurement
(3) sample size, \( n \)
(4) measure of variability, e.g., \( s, s^2, s_{\bar{X}}, 95\% \text{ CI} \)

7 Combining Means

7.1 In general, \( \mu[f(X,Y)] \neq f[\mu(X),\mu(Y)]; \quad \sigma[f(X,Y)] \neq f[\sigma(X),\sigma(Y)] \)

7.2 sum of random variables:
\[
\mu(X + Y) = \mu(X) + \mu(Y) \quad \sigma(X + Y)^2 = \sigma(X)^2 + \sigma(Y)^2 + 2\text{cov}(X,Y)
\]

7.3 product of random variables:
\[
\mu(XY) = \mu(X)\mu(Y) + \text{cov}(X,Y)
\]
if \( X \) and \( Y \) are independent, \( \mu(XY) = \mu(X)\mu(Y) \)

7.4 ratio of random variables:
\[
\mu(X / Y) = \mu(X) / \mu(Y) - \text{cov}(X / Y,Y) / \mu(Y)
\]