1. Determine the edge chromatic number of $K_{m,n}$.
2. Prove that a regular graph of degree 5 cannot be decomposed into subgraphs, each isomorphic to a path of length 6.
3. Let $\bar{G}$ denote the complement of $G$ (on the same vertex set as $G$). Show that
   
   $\chi(G) + \chi(\bar{G}) \geq 2\sqrt{n}.$

4. Show that a graph of order $n$ and size $(k - 1)n - \binom{k}{2} + 1$ contains every tree of order $k + 1$.
5. Show that a graph with $n$ vertices and minimum degree $\lceil \frac{(r-2)n}{r-1} \rceil + 1$ contains a $K_r$.
6. Let $x_1, x_2, \ldots, x_n \in \mathbb{R}^d$ for some $d$. Suppose that $||x_i|| = 1$ for all $i$. Prove that there are at most $\lfloor n^2/4 \rfloor$ unordered pairs $i, j$ such that $||x_i + x_j|| < 1$.
   
   [Hint. Show that $||x_i + x_j|| \geq 1$ for some $1 \leq i < j \leq 3$.]

7. Prove that the Ramsey number $R(3, 4)$ is 9.
8*. By considering the graph on the integers modulo 17 in which $i$ is joined to $j$ iff $i - j$ is a square mod 17 (i.e. 1, 2, 4, 8, 9, 13, 15 or 16), prove that $R(4, 4) = 18$. 