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Galois representations with open image

Suppose that $p$ is a prime and that $n \geq 1$. Let $G_{\mathbb{Q}} = Gal(\overline{\mathbb{Q}}/\mathbb{Q})$ be the absolute Galois group of $\mathbb{Q}$. Let $\mathbb{Z}_p$ denote the ring of $p$-adic integers. Our purpose in this talk is to describe a way of constructing continuous representations

$$\rho : G_{\mathbb{Q}} \longrightarrow GL_n(\mathbb{Z}_p)$$

whose image is open. This means that the image of $\rho$ has finite index in $GL_n(\mathbb{Z}_p)$. We can do this for many pairs $(n,p)$. One typical result is the following:

Proposition: Suppose that $p$ is a regular prime and that $p \geq 4\left\lceil \frac{n}{2} \right\rceil + 1$. Then there exists a continuous representation $\rho$ as above with open image.