Math 240
Practice Final Exam

Note: Show work whenever possible to receive full credit.

1. Determine which of the four levels of measurements (nominal, ordinal, interval, ratio) is most appropriate. [4½ pts]
   a) Amount of monthly electricity consumption in a household.

   Ans:……………………………...

   b) Survey responses of “good, better, best.”

   Ans:……………………………...

   c) Arrival time (in hours and minutes) of a WTA bus.

   Ans:……………………………...

2. Identify which of these types of sampling is used: random, stratified, systematic, cluster, convenience. [4½ pts]
   a) A tax auditor selects every 500th income tax return that is received.

   Ans:……………………………...

   b) A pollster uses a computer to generate 500 random numbers, then interviews the voters corresponding to those numbers.

   Ans:……………………………...

   c) An education researcher randomly selects 48 middle schools and interviews all the teachers at each school.

   Ans:……………………………...

3. In a collegiate conference, student athletes of a particular sport are divided into three groups based on their weight: light, average, heavy. Assume that each group has equal number of members. An official obtains a sample of athletes by selecting 5 athletes from each group. [4 pts]
   a) Does this sampling plan result in a simple random sample (SRS)? Yes No

   b) Does this sampling plan result in a random sample? Yes No
4. Suppose a population has bell-shaped distribution with some mean and standard deviation. What percentage of the population
a) is within one standard deviation of the mean? [1 pt]

Ans:…………………….

b) exceeds one standard deviation above the mean? [2 pts]

Ans:…………………….

5. In American roulette, the wheel has the 38 numbers

00, 0, 1, 2, . . . , 34, 35, and 36

marked on equally spaced slots. If a player bets $1 of her own money on a number and wins, then the player keeps the dollar and receives an additional $35. Otherwise, the dollar is lost. Find the expected net winnings of a player in this game. [4 pts]

Ans:…………………….

6. According to Stanford School of Medicine, 36% of Americans are blood type A-positive. Suppose four people in the U.S. are randomly chosen.

a) Find the probability that all four are blood type A-positive. [3 pts]

Ans:…………………….

b) Find the probability that at least one of the four is blood type A-positive. [3 pts]

Ans:…………………….
7. The national average SAT score (for both Verbal and Math) is 1028. Assume a normal distribution for the scores with \( \sigma = 92 \).
   a) What is the probability that a randomly selected score exceeds 1200?

   Ans:…………………………..

   b) What is the 25\(^{th}\) percentile of the scores? [3 pts]

   Ans:…………………………..

8. For each of the following, state whether the statement is true or false (T/F). [7½ pts]
   a) ____ The significance level \( \alpha \) in a hypothesis test is the probability of making a Type I error.

   b) ____ The probability that \( \text{none} \) of four randomly chosen people will have the same birthday is 0.984 (to the nearest 3 decimal places).

   c) ____ We should use the \( t \) distribution when computing a confidence interval for the following: \( n = 12, \sigma \) is unknown, and population is very skewed.

   d) ____ For a 95\% confidence interval, the smaller the sample size, the narrower the confidence interval.

   e) ____ The contingency table below representing the outcomes of two groups of patients suffering from a particular ailment, one receiving Drug A and the other Drug B. Assuming that the contingency table represents the entire \( \text{population} \) of patients, it shows that the type of drug and the change in their condition are independent. (Note: Hypothesis test is not needed here.)

<table>
<thead>
<tr>
<th></th>
<th>Improved</th>
<th>No change</th>
<th>Deteriorated</th>
</tr>
</thead>
<tbody>
<tr>
<td>Drug A</td>
<td>30</td>
<td>12</td>
<td>18</td>
</tr>
<tr>
<td>Drug B</td>
<td>45</td>
<td>18</td>
<td>27</td>
</tr>
</tbody>
</table>
9. In a hypothesis test for \( H_0 : p = 0.25 \) vs. \( H_1 : p \neq 0.25 \), the test statistic for the data turned out to be 2.72, i.e., \( z = 2.72 \). Find the exact \( p \)-value and state whether you can reject \( H_0 \) at a significance level of 0.01. [3 pts]

\[ p\text{-value:} \]

Circle one: Reject \( H_0 \) Don’t reject \( H_0 \)

10. Consider a hypothesis test for the population proportion \( H_0 : p = 0.60 \) vs. \( H_1 : p \neq 0.60 \). Suppose the true population proportion \( p \) is 0.55 (i.e., the null hypothesis is false). If the sample somehow, by statistical fluke, gives a test statistic that does not fall in the critical region, we will not reject \( H_0 \), thus resulting in a wrong conclusion. What is the name given to such an error in conclusion? [1½ pts]

Ans: ……………………………………….

11. A sample of 6 college wrestlers had an average weight of 276 pounds with a sample standard deviation of 12 pounds. Assume the population of weights of college wrestlers has a normal distribution and the true population standard deviation is unknown.

a) Construct a 90% confidence interval for the true mean weight of all college wrestlers. [4 pts]

Ans: ……………………………………….

b) Interpret the confidence interval. [2 pts]

Ans: ……………………………………….

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12. A researcher is interested in the results concerning the amount of time spent in volunteer service per week among college students. She collected a random sample of 20 female college students and 18 male college students that includes the information about their volunteer service hours per week. The summary of the data are as follow:

<table>
<thead>
<tr>
<th>Gender</th>
<th>Sample size</th>
<th>Mean</th>
<th>Standard deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Male</td>
<td>18</td>
<td>2.5</td>
<td>2.2</td>
</tr>
<tr>
<td>Female</td>
<td>20</td>
<td>3.8</td>
<td>3.5</td>
</tr>
</tbody>
</table>

Test at the 0.01 significance level the claim that the mean volunteer hours per week for male students is less than that for female students. [10 pts]

Null hypothesis $H_0$ : ……………………

Alternative hypothesis $H_1$ : ………………

Value of test statistic:………………… …

Critical value(s):…………………

Reject $H_0$? (Circle one): Yes No

Conclusion (in nontechnical terms):……………………………………………………………………
…………………………………………………………………………………………………………
…………………………………………………………………………………………………………
…………………………………………………………………………………………………………

13. For the hypothesis test in the previous question, give the $p$-value or the best bound(s) for the $p$-value, whichever is more appropriate. [3 pts]

Ans:……………………………..
14. The tensile strength of a metal is a measure of its ability to resist tearing when it is pulled lengthwise. The tensile strengths of two types of steel, labeled 1 and 2, were compared using randomly chosen steel bars from each type. The results were given in the following SPSS output below.

The claim to be tested is that the mean tensile strength of the type 1 is greater than that of type 2 at the significance level \( \alpha = 0.01 \). Note: We will assume that the true variances of the tensile strength of both types of steel are not equal.

![SPSS output]

**Group Statistics**

<table>
<thead>
<tr>
<th>method</th>
<th>N</th>
<th>Mean</th>
<th>Std. Deviation</th>
<th>Std. Error Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>tensile 1.00</td>
<td>12</td>
<td>389.9167</td>
<td>15.09038</td>
<td>4.35622</td>
</tr>
<tr>
<td>2.00</td>
<td>10</td>
<td>378.1000</td>
<td>16.59618</td>
<td>5.24817</td>
</tr>
</tbody>
</table>

**Independent Samples Test**

<table>
<thead>
<tr>
<th></th>
<th>t</th>
<th>df</th>
<th>Sig. (2-tailed)</th>
<th>Mean Difference</th>
<th>Std. Error Difference</th>
<th>Upper</th>
<th>Lower</th>
</tr>
</thead>
<tbody>
<tr>
<td>tensile</td>
<td>1.748</td>
<td>20</td>
<td>.096</td>
<td>11.81667</td>
<td>6.75907</td>
<td>-2.28250</td>
<td>25.91583</td>
</tr>
<tr>
<td>Equal variances assumed</td>
<td>Equal variances not assumed</td>
<td>1.733</td>
<td>18.492</td>
<td>.100</td>
<td>11.81667</td>
<td>6.82056</td>
<td>-2.48552</td>
</tr>
</tbody>
</table>

Based on the output, fill out the following: [8 pts]

Null hypothesis \( H_0 \): ..............................

Alternative hypothesis \( H_1 \): ..............................

Value of test statistic: ................. ...

\( p \)-value: ..................

Reject \( H_0 \)? (Circle one): Yes No

Conclusion: .................................................................
15. A psychologist was interested in whether type of conditioning stimulus is independent of the type of mouse. One hundred mice were used in the study. The following table shows the number of each type of mice that completed their assigned task satisfactorily, given the type of stimulus.

<table>
<thead>
<tr>
<th>Type of mouse</th>
<th>Classical</th>
<th>Operant</th>
</tr>
</thead>
<tbody>
<tr>
<td>White</td>
<td>22</td>
<td>38</td>
</tr>
<tr>
<td>Brown</td>
<td>12</td>
<td>28</td>
</tr>
</tbody>
</table>

Test the claim that the type of mouse and the type of stimulus are independent. Use $\alpha = 0.01$.

[10 pts]

Null hypothesis $H_0$:

Alternative hypothesis $H_1$:

Value of test statistic: 

Critical value:

Reject $H_0$? (Circle one): Yes No

Conclusion (in nontechnical terms): 

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16. A study was done to study the effects of ethanol on sleep time. Four different concentrations of ethanol were used (0, 1 g/kg, 2 g/kg, 4 g/kg). A random sample of 20 rats, matched for age and other characteristics, was selected and divided into 4 groups of five. Each group was randomly assigned to one of the four different concentrations, and each rat in the group was given an injection having the particular concentration of ethanol assigned to it. The REM sleep time of each rat was recorded for a 24-hour period, and the results were used to construct the ANOVA table below.

<table>
<thead>
<tr>
<th>Source</th>
<th>SS</th>
<th>df</th>
<th>MS</th>
<th>F</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>Treatment</td>
<td>5882.4</td>
<td></td>
<td></td>
<td>0.0001</td>
<td></td>
</tr>
<tr>
<td>Error</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>7369.8</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

a) What are the “treatments”? [2 pts]

Ans: ………………………………………………….

b) Complete the ANOVA table. [4 pts]

c) The ANOVA table is used to do a hypothesis of the equality of means. Give the name of the distribution needed to do the test (as an example, for the test of independence the chi-square distribution is needed). [2 pts]

Ans: ………………………………………………….

d) Based on the ANOVA table, test the claim that the mean REM sleep times for the four different concentrations are the same. Use \( \alpha = 0.05 \). [8 pts]

Null hypothesis \( H_0 : \) ………………………………………………….

Alternative hypothesis \( H_1 : \) ………………………………………………….

Value of test statistic: ………………

\( p \)-value: …………………

Reject \( H_0 \)? (Circle one): Yes No

Conclusion (in nontechnical terms): ………………………………………………….
……………………………………………….
……………………………………………….
……………………………………………….
Answers

1. (a) ratio  (b) ordinal  (c) interval
2. (a) systematic  (b) random  (c) cluster
3. (a) No  (b) Yes
4. (a) 68%  (b) 16%
5. -$0.0526
6. (a) 0.0168  (b) 0.8322
7. (a) 0.0307  (b) 965.9
8. (a) T  (b) T  (c) F  (d) F  (e) T
9. 0.0066, reject $H_0$
10. Type II error
11. (a) (266.13, 285.87)  (b) We are 90% confident that the true mean weight of college wrestlers is between 266.13 and 285.87 lbs.
12. $H_0 : \mu_m = \mu_f$,  $H_1 : \mu_m < \mu_f$, t.s. = -1.38, c.v. = -2.567, do not reject $H_0$, there is not enough evidence to support the claim that the mean volunteer hours per week for male students is less than that for female students.
13. 0.05 < p-value < 0.10
14. $H_0 : \mu_1 = \mu_2$,  $H_1 : \mu_1 > \mu_2$, t.s. = 1.733, p-value = 0.05, do not reject $H_0$, there is not enough evidence to support the claim that the mean tensile strength of Type 1 is greater than that of Type 2.
15. $H_0 :$ type of mouse and type of stimulus are independent (claim),  $H_1 :$ the two variables are not independent, t.s. = 0.475, c.v. = 6.635, do not reject $H_0$, there is not enough evidence to warrant rejection of the claim that the type of stimulus and the type of mouse are independent.
16. (a) different concentrations of ethanol  (c) F distribution  
   (d) $H_0 : \mu_1 = \mu_2 = \mu_3 = \mu_4$,  $H_1 :$ not all the means are equal, t.s. = 21.09, p-value = 0.0001, reject $H_0$, there is enough evidence to warrant rejection of the claim that the mean REM sleep times of the 4 different concentrations are the same.