Chapter 6 Practice Problems

1. Consider a 1 year discount bond with a face value of $1000. Suppose the demand for this bond is given by the equation \( P = 1100 - B \) where \( P \) is the price of the bond and \( B \) is the number of bonds demanded. Suppose the supply for this bond is given by the equation \( P = 100 + 4B \).

a. What is the equilibrium price for this bond? What is the equilibrium interest rate on bonds? What is the equilibrium number of bonds supplied?

Setting the supply and demand equations equal to one another gives an equilibrium quantity of bonds as \( B = 200 \) and price as \( P = 900 \). Using the present value equation to solve for the equilibrium interest rate gives \( 900 = 1000/(1+i) \) so \( i = .1111 \) or 11.11%.

b. Suppose that the President, claiming to save social security, orders the Treasury to create and sell more bonds. After this, the supply of bonds increases to \( P = 50 + 4B \). What is the new equilibrium level of interest rates, bond prices, and bonds traded?

Again, setting the supply and demand equations equal to one another gives an equilibrium quantity of bonds as \( B = 210 \) and price as \( P = 890 \). Using the present value equation to solve for the equilibrium interest rate gives \( 890 = 1000/(1+i) \) so \( i = .1235 \) or 12.35%.

2. Imagine that companies and the government borrow on the bond market using 10 year discount bonds with a face value of $1000. Total demand for these bonds are given by the equation \( P = 1050 - .5B \) where \( P \) is the price of the bond and \( B \) is the number of bonds demanded. Suppose the government supplies bonds according to the equation \( P = B^G \) and private companies supply bonds according to the equation \( P = 2B^C \) where \( B^G \) is the number of government bonds issued and \( B^C \) is the number of corporate bonds issued. \( B^G + B^C \) is then the total amount of bonds issued (\( B \)). Assume companies are as risky as the government so the price of government bonds and corporate bonds are equal.

a. What is the equilibrium price of bonds in this example? What is the equilibrium interest rate of bonds? How many bonds does the government sell? How many bonds do corporations sell?

Total supply of bonds is equal to \( P = (2/3)B \) (remember, you add up supply and demand curves horizontally, not vertically). Setting this supply curve equal to the demand curve gives a total amount of bonds sold as 900 and the equilibrium price of bonds as 600. Of these 900, 600 are government bonds and 300 are corporate bonds. The equilibrium interest rate solves the present value formula \( 600 = 1000/(1+i)^{10} \). \( i = .052 \) or \( i = 5.2\% \) solves this.

b. Imagine that the government increased its borrowing (to “stimulate” the economy). In doing so, its new bond supply curve is \( P = B^G - 35 \). What is the equilibrium price of bonds? What is the equilibrium interest rate? How many bonds does the government sell? How many bonds do corporations sell?

The total supply of bonds is equal to \( P = (2/3)B - 70/3 \). Setting this supply curve equal to the demand for bonds gives \( B = 920 \) and the \( P = 590 \). The equilibrium \( i \) is .0541 or \( i = 5.41\% \). Notice, as the government borrows more, the price of bonds fall and the interest rate rises. In this case \( B^G = 625 \) and \( B^C = 295 \).
c. Is the government’s object of stimulating the economy in part b relative to part a successful? Comment.

In part a 900 bonds were sold at $600 a piece raising $540,000 of which $180,000 (300×600) would fund economic investment and $360,000 (600×600) for increased government purchases. In part b 920 bonds were sold at $590 a piece raising $542,800 of which $174,050 funds economic investment and $368,750 funds government purchases. A few notes:

1. The total amount of borrowing increases in part b suggesting that more purchases are able to be made when the government increases its borrowing. Of course, this ignores the fact that lenders are not spending this money. In other words, without the more attractive bonds (which pay 5.41% annual interest instead of 5.2%), some savers may have chosen to make purchases which stimulated the economy. So, given the information provided, we cannot know if the economy is stimulated or if the same amount of total purchases were made and just the entity making those purchases changed (government v. households).

2. The increased supply of government bonds “crowded out” sales of corporate bonds. This occurs because the increased supply of government bonds reduces the equilibrium price of bonds and makes selling bonds less appealing to corporations (after all, they now have to pay a higher interest rate as a result of more government borrowing). If economic investment has a higher social return than government purchases, then society would be worse off as a result (i.e. businesses make productive capital investments while the government buys things of little value to society). Of course, it is possible that government provides socially valuable goods (e.g. public goods) and this added borrowing increases social welfare beyond what would have occurred had the private companies been willing to borrow.

3. Consider a one year, 10-percent coupon bond with a safe value of $1,000 issued by a private corporation. The one year risk-free rate is 10 percent. The corporation has hit on hard times and the consensus is that there is a 20 percent probability that it will default on its bonds. If an investor were willing to pay $775 for the bond, is that investor risk neutral, risk loving, or risk averse?

If the bond were risk free, it would pay off $1100 in one year’s time - $100 coupon payment and $1000 face value of the bond. If there is a 20% risk of default, then the expected value of these payment flows associated with the bond are $1100 *.8 + 0*.2 = $880. The present value of $880 in one year’s time is $880/1.1 = $800. This would be the price a risk-neutral investor would be willing to pay. If the investor is only willing to pay $775 for the bond, he or she requires compensation for bearing the risk associated with the bond and so is risk averse.

4. Use the model of supply and demand for bonds to illustrate and explain the impact of each of the following on the equilibrium quantity and price of bonds.
   a. A new company forms which specializes in helping investors trade bonds easily and inexpensively.
      Demand for bonds increase.
   b. Expected inflation falls invoking a stronger response from issues of bonds than from purchasers of bonds.
      Demand for bonds increases. Supply for bonds falls. The supply falls further than the increase in the demand for bonds.
c. The government increases the capital gains tax and uses that tax to increase education expenditures (thus not impacting the government budget surplus/deficit).  
Demand for bonds decrease (as their after-tax profit is now lower).

d. All leading economic indicators suggest stronger economic growth in the near future.  
Supply of bonds increases.

e. Brokerage commissions on stocks fall.  
Demand for bonds decreases.

5. It has been long observed that interest rates are procyclical (meaning interest rates rise when the economy is expanding and falling when the economy is contracting).  Explain why this occurs.  
During economic expansions, firms face higher demand for their product and therefore hope to expand their operations to meet this increased demand.  In order to do so, firms need more capital and finance purchases of that capital through borrowing (selling bonds).  As the supply of bonds increases, bond prices fall and interest rates rise.

This impact is mitigate some by increased household saving that occurs during expansions.  As household savings rises, households purchase more bonds thus raising their price and causing interest rates to fall.  However, household savings does not rise as fast during expansions as does the firm’s desire to acquire borrowed funds.  As the economy expands, households also increase their borrowing (on the hypothesis that they will be able to repay these loans during continued good economic times).  This increase in household borrowing reduces total household net savings and therefore reduces the upward pressure on interest rates that would otherwise occur.

6. How might a sudden increase in the people’s expectations of future real estate prices affect interest rates?  
Interest rates would rise.  A sudden increase in people’s expectations of future real estate prices raises the expected return on real estate relative to bonds, so the demand for bonds falls.  As the demand curve for bonds falls, the equilibrium price fall and the equilibrium interest rate rises.